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Schwinger-boson mean-field method for an alternating-spin (1/2, 1) two-leg ladder

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Abstract
We study the low-lying excitations and thermodynamic properties of a ferrimagnetic Heisenberg two-leg ladder, which is composed of alternating-spin (1/2, 1) double chains with antiferromagnetic coupling between the chains. The spin excitation spectra as well as the corresponding energy gaps are calculated by means of the Schwinger-boson mean-field theory. The two interactions between the chains and along the chains play an important role in determining the two branches of the spectrum, not only regarding the size of the low-lying excitation energy gap but also the bandgap between the optical spectrum and the acoustic spectrum. In addition, the two energy gaps and the specific heat have also been calculated at low temperatures.

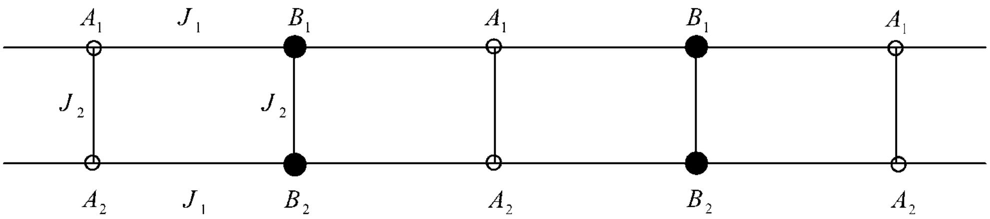
1. Introduction

A variety of exotic physical phenomena in low-dimensional magnetic systems, such as Haldane gap systems, have been attracting much interest in recent years. Owing to strong quantum fluctuations, these quantal many-body systems produce a variety of interesting phenomena. Haldane conjectured that a one-dimensional spin chain is gapful for integer spin but gapless for a half-odd-integer spin chain [1]. This has been demonstrated both experimentally and theoretically. Current interest has spread to wider classes of spin ladders, stimulated by the experimental realization of a variety of spin systems [2]. These spin ladders consist of coupled one-dimensional chains, which can be divided into uniform-spin ladders and alternating-spin ladders. Theoretical studies [3] have suggested that there are two different universal classes for uniform-spin ladders, i.e. the antiferromagnetic spin-1/2 ladders are gapful or gapless depending on whether n (the number of legs) is even or odd [1, 4]. These predictions have been confirmed experimentally on $LaCuO_{2.5}$ [5] and $(VO)_2P_2O_7$ [6].

Similar behaviours have also been found in ferrimagnetic Heisenberg ladders. It was predicted by the spin wave mean-field theory (SWMFT) that an interplay between two ferrimagnetic Heisenberg chains would send the double chains into a new disordered phase, which was proved by the nonlinear sigma model (NLSM) [7]. Also, the results for the density matrix renormalization group (DMRG) further prove that any positive interchain coupling drives the system to a gapped ground state [8]. However, up to now corresponding analytical work has been relatively limited, especially in comparison with work on uncoupled ferrimagnetic chains. In a ferrimagnetic Heisenberg two-leg ladder, two energy scales control the excitation spectrum: the intrachain exchange constant J_1 and the transverse interchain exchange coupling J_2. Consequently, we plan to discuss the excitation spectrum in such a two-leg mixed-spin ladder system composed of two ferrimagnetic Heisenberg chains with two kinds of spin $S^A = 1/2$ and $S^B = 1$. This 'ferrimagnetic Heisenberg two-leg ladder' has isotropic couplings J_1 along the chains and J_2 between them. The Hamiltonian of this model is represented by:

$$H = J_1 \sum \left(S_{2i}^{A_1} S_{2i+1}^{B_1} + S_{2i}^{A_2} S_{2i+1}^{B_2}\right) + J_2 \sum \left(S_{2i}^{A_1} S_{2i}^{A_2} + S_{2i+1}^{B_1} S_{2i+1}^{B_2}\right), \tag{1}$$

and its configuration is graphed as follows:



We choose four nearest-neighbour sublattices ($S^{A_1} = S^A, S^{A_2} = S^A$ and $S^{B_1} = S^B, S^{B_2} = S^B$) as a unit cell. The system contains N such unit cells. Here we limit our discussions to the range of $J_i > 0$ $(i = 1, 2)$.

SWMFT, in dealing with the low-lying excitation spectrum, was used in a one-dimensional spin (1, 1/2) ferrimagnetic Heisenberg chain and gave results which were in good agreement with those from the DMRG [9]. But for the two-leg ladder system, SWMFT is invalid for predicting a possible opening of a spin gap due to interaction between chains [7]. Here, we study this two-leg ladder by means of the Schwinger-boson mean-field theory (SBMFT). Of the analytical methods, the Schwinger-boson approach proposed by Auerbach and Arovas is the one of the most successful [10, 11]. Contrary to the SWMFT approach, the SBMFT approach does not rely on a magnetized ground state, which enables it to describe both ordered and disordered phases on an equal footing. It has been successfully applied in a one-dimensional ferrimagnetic Heisenberg chain [12] and a two-dimensional mixed-spin model on a square lattice [13]. Therefore, we may go on to employ it in this 'in between' case, between one dimension and two dimensions of a 'ferrimagnetic Heisenberg two-leg ladder'. We find that the Schwinger-boson approach is suitable for describing both the gapful ground state and the thermodynamic properties at low temperatures. The energy gap of the low-lying spectrum derived from SBMFT is in good agreement with that from DMRG by Trumper and Gazza [8].

In our SBMFT approach, there are two branches of the gapful excitation spectrum for any finite coupling parameters J_1 and J_2 in the ladder system. The proportions of J_1 and J_2 determine the shape and gap of the excitation spectra. The two gaps at $T = 0$ K are calculated for different J_1 and J_2. The two gaps and the specific heat at low temperatures are also calculated for a special gapful case of $J_1 = J_2 = 1$. An exponential law is observed for the specific heat near $T = 0$ K because of a non-zero gap in the acoustic spectrum at $T = 0$ K.

This paper is organized as follows. In section 2 we introduce briefly the Schwinger-boson techniques. The properties of the excitation spectrum of the ground state at $T = 0$ K and at low temperatures are discussed in section 3. A brief summary is given at the end.

2. Schwinger-boson mean field theory

The four spin operators $S_1^{A_n}$ and $S_1^{B_n}$ in equation (1) can be represented by eight kinds of Schwinger bosons $(a_{l,\uparrow}^{(1)}a_{l,\downarrow}^{(1)}a_{l,\uparrow}^{(2)}a_{l,\downarrow}^{(2)})$, and $(b_{l,\uparrow}^{(1)}b_{l,\downarrow}^{(1)}b_{l,\uparrow}^{(2)}b_{l,\downarrow}^{(2)})$ on their sublattices,

$$S_{l,+}^{A_n} = a_{l,\uparrow}^{(n)+}a_{l,\downarrow}^{(n)} \qquad S_{l,z}^{A_n} = \tfrac{1}{2}(a_{l,\uparrow}^{(n)+}a_{l,\uparrow}^{(n)} - a_{l,\downarrow}^{(n)+}a_{l,\downarrow}^{(n)}) \tag{2}$$

$$S_{l,+}^{B_n} = b_{l,\uparrow}^{(n)+}b_{l,\downarrow}^{(n)} \qquad S_{l,z}^{B_n} = \tfrac{1}{2}(b_{l,\uparrow}^{(n)+}b_{l,\uparrow}^{(n)} - b_{l,\downarrow}^{(n)+}b_{l,\downarrow}^{(n)}), \tag{3}$$

where $n = 1$ and 2 represent the top chain and the bottom chain, respectively. On each sublattice there are constraining conditions: $a_{l,\uparrow}^{(n)+}a_{l,\uparrow}^{(n)} + a_{l,\downarrow}^{(n)+}a_{l,\downarrow}^{(n)} = 2S^A$ (for $l = 2i$) and $b_{l,\uparrow}^{(n)+}b_{l,\uparrow}^{(n)} + b_{l,\downarrow}^{(n)+}b_{l,\downarrow}^{(n)} = 2S^B$ (for $l = 2i + 1$). By imposing the constraints on each site, we can correctly map the original spin system to the bosonic system. Introducing four Lagrange multipliers λ_{A_n} and λ_{B_n} for sublattices A_n and B_n respectively we can obtain the mean field Hamiltonian after the Fourier transformation:

$$\begin{aligned}
H^{MF} = {} & 4NJ_1S^AS^B + NJ_2S^AS^A + NJ_2S^BS^B + 8NJ_1A^2 \\
& + 2NJ_2C^2 + 2NJ_2D^2 - 4N\lambda_AS^A - 4N\lambda_BS^B \\
& + \lambda_A\sum_k\left[a_{k,\uparrow}^{(1)+}a_{k,\uparrow}^{(1)} + a_{k,\downarrow}^{(1)+}a_{k,\downarrow}^{(1)} + a_{k,\uparrow}^{(2)+}a_{k,\uparrow}^{(2)} + a_{k,\downarrow}^{(2)+}a_{k,\downarrow}^{(2)}\right] \\
& + \lambda_B\sum_k\left[b_{k,\uparrow}^{(1)+}b_{k,\uparrow}^{(1)} + b_{k,\downarrow}^{(1)+}b_{k,\downarrow}^{(1)} + b_{k,\uparrow}^{(2)+}b_{k,\uparrow}^{(2)} + b_{k,\downarrow}^{(2)+}b_{k,\downarrow}^{(2)}\right] \\
& - J_1\sum_k\left[z\gamma_kA\mathrm{e}^{-\mathrm{i}\theta_A}(a_{k,\uparrow}^{(1)}b_{k,\downarrow}^{(1)} - a_{k,\downarrow}^{(1)}b_{k,\uparrow}^{(1)}) + z\gamma_kA\mathrm{e}^{\mathrm{i}\theta_A}(a_{k,\uparrow}^{(1)+}b_{k,\downarrow}^{(1)+} - a_{k,\downarrow}^{(1)+}b_{k,\uparrow}^{(1)+})\right] \\
& - J_1\sum_k\left[z\gamma_kA\mathrm{e}^{-\mathrm{i}\theta_B}(a_{k,\uparrow}^{(2)}b_{k,\downarrow}^{(2)} - a_{k,\downarrow}^{(2)}b_{k,\uparrow}^{(2)}) + z\gamma_kA\mathrm{e}^{\mathrm{i}\theta_B}(a_{k,\uparrow}^{(2)+}b_{k,\downarrow}^{(2)+} - a_{k,\downarrow}^{(2)+}b_{k,\uparrow}^{(2)+})\right] \\
& - J_2\sum_k\left[C\mathrm{e}^{-\mathrm{i}\theta_C}(a_{k,\uparrow}^{(1)}a_{k,\downarrow}^{(2)} - a_{k,\downarrow}^{(1)}a_{k,\uparrow}^{(2)}) + C\mathrm{e}^{\mathrm{i}\theta_C}(a_{k,\uparrow}^{(1)+}a_{k,\downarrow}^{(2)+} - a_{k,\downarrow}^{(1)+}a_{k,\uparrow}^{(2)+})\right] \\
& - J_2\sum_k\left[D\mathrm{e}^{-\mathrm{i}\theta_D}(b_{k,\uparrow}^{(1)}b_{k,\downarrow}^{(2)} - b_{k,\downarrow}^{(1)}b_{k,\uparrow}^{(2)}) + D\mathrm{e}^{\mathrm{i}\theta_D}(b_{k,\uparrow}^{(1)+}b_{k,\downarrow}^{(2)+} - b_{k,\downarrow}^{(1)+}b_{k,\uparrow}^{(2)+})\right],
\end{aligned} \tag{4}$$

where $\sum_k$ means the sum of k over the first Brillouin zone. The structure factor γ_k is defined as $\gamma_k = \frac{1}{z}\sum_{\eta=\pm1}\mathrm{e}^{\mathrm{i}\eta k}$. z is the number of nearest neighbours and η denotes the nearest-neighbour site. The four bond operators we used in the above equation are introduced as:

$$\begin{aligned}
A_{2i,2i+\eta} &= \tfrac{1}{2}(a_{2i,\uparrow}^{(1)}b_{2i+\eta,\downarrow}^{(1)} - a_{2i,\downarrow}^{(1)}b_{2i+\eta,\uparrow}^{(1)}) \\
B_{2i,2i+\eta} &= \tfrac{1}{2}(a_{2i,\uparrow}^{(2)}b_{2i+\eta,\downarrow}^{(2)} - a_{2i,\downarrow}^{(2)}b_{2i+\eta,\uparrow}^{(2)}) \\
C_{2i} &= \tfrac{1}{2}(a_{2i,\uparrow}^{(1)}a_{2i,\downarrow}^{(2)} - a_{2i,\downarrow}^{(1)}a_{2i,\uparrow}^{(2)}) \\
D_{2i+1} &= \tfrac{1}{2}(b_{2i+1,\uparrow}^{(1)}b_{2i+1,\downarrow}^{(2)} - b_{2i+1,\downarrow}^{(1)}b_{2i+1,\uparrow}^{(2)}).
\end{aligned} \tag{5}$$

In equation (4), we have considered the symmetry between the top chain and the bottom chain in the ladder: $\langle A_{2i,2i+1}\rangle = \langle B_{2i,2i+1}\rangle = A\mathrm{e}^{\mathrm{i}\theta_A}$, $\langle C_{2i}\rangle = C\mathrm{e}^{\mathrm{i}\theta_C}$ and $\langle D_{2i+1}\rangle = D\mathrm{e}^{\mathrm{i}\theta_D}$. Here A, C and D are the real amplitudes, and θ_A, θ_C and θ_D the corresponding phase factors. Since $\lambda_{A_1} = \lambda_{A_2}$ and $\lambda_{B_1} = \lambda_{B_2}$ for the same reason of symmetry, we can rename them λ_A and λ_B respectively.

By diagonalizing the mean-field Hamiltonian via the Bogliubov transformation, we have

$$\begin{aligned} H^{MF} = E_{const} &+ \sum_k \left[E^-(k)(\alpha^{(1)+}_{k,\uparrow}\alpha^{(1)}_{k,\uparrow} + \beta^{(1)+}_{k,\downarrow}\beta^{(1)}_{k,\downarrow} + 1 + \beta^{(2)+}_{k,\uparrow}\beta^{(2)}_{k,\uparrow} + \alpha^{(2)+}_{k,\downarrow}\alpha^{(2)}_{k,\downarrow} + 1)\right] \\ &+ \sum_k \left[E^+(k)(\alpha^{(1)+}_{k,\downarrow}\alpha^{(1)}_{k,\downarrow} + \beta^{(1)+}_{k,\uparrow}\beta^{(1)}_{k,\uparrow} + 1 + \beta^{(2)+}_{k,\downarrow}\beta^{(2)}_{k,\downarrow} + \alpha^{(2)+}_{k,\uparrow}\alpha^{(2)}_{k,\uparrow} + 1)\right] \\ &= E_{const} + 2\sum_k \left[E^-(k)(2n^-_k + 1) + E^+(k)(2n^+_k + 1)\right], \end{aligned} \tag{6}$$

with

$$\begin{aligned} E_{const} = &\, 4NJ_1S^AS^B + NJ_2S^AS^A + NJ_2S^BS^B + 8NJ_1A^2 \\ &+ 2NJ_2C^2 + 2NJ_2D^2 - 2N\lambda_A(2S^A + 1) - 2N\lambda_B(2S^B + 1). \end{aligned}$$

For each k, there are eight branches of spectra, made up of two groups of four each having the same excitation energy. They can be divided into two classes: one belonging to the acoustic branch $E^-(k)$, the other belonging to the optical branch $E^+(k)$. From a statistical point of view, we have $\langle\alpha^{(1)+}_{k,\uparrow}\alpha^{(1)}_{k,\uparrow}\rangle = \langle\beta^{(1)+}_{k,\downarrow}\beta^{(1)}_{k,\downarrow}\rangle = \langle\beta^{(2)+}_{k,\uparrow}\beta^{(2)}_{k,\uparrow}\rangle = \langle\alpha^{(2)+}_{k,\downarrow}\alpha^{(2)}_{k,\downarrow}\rangle$ and $\langle\alpha^{(1)+}_{k,\downarrow}\alpha^{(1)}_{k,\downarrow}\rangle = \langle\beta^{(1)+}_{k,\uparrow}\beta^{(1)}_{k,\uparrow}\rangle = \langle\beta^{(2)+}_{k,\downarrow}\beta^{(2)}_{k,\downarrow}\rangle = \langle\alpha^{(2)+}_{k,\uparrow}\alpha^{(2)}_{k,\uparrow}\rangle$. Therefore, we can rewrite them as n^-_k and n^+_k, respectively, which are the Bose-type quasi-particles with energies $E^-(k)$ and $E^+(k)$. The corresponding excitation spectra are:

$$E^-(k) = \sqrt{\frac{E_0 - \sqrt{E_1}}{2}}, \qquad E^+(k) = \sqrt{\frac{E_0 + \sqrt{E_1}}{2}}, \tag{7}$$

$$\begin{aligned} E_0 &= \lambda_A^2 + \lambda_B^2 - 2(2J_1A\cos[k])^2 - (J_2C)^2 - (J_2D)^2 \\ E_1 &= (\lambda_A^2 - \lambda_B^2 - (J_2C)^2 + (J_2D)^2) - 4(2J_1A\cos[k])^2((\lambda_A - \lambda_B)^2 - (J_2C)^2 \\ &\quad - (J_2D)^2 - 2J_2^2CD\cos[2\theta_A - \theta_C - \theta_D]). \end{aligned} \tag{8}$$

At the same time, we define the two energy gaps as follows:

$$\Delta^- = 2\min[E^-(k)], \qquad \Delta^+ = 2\min[E^+(k)], \tag{9}$$

where Δ^- and Δ^+ are the energy gaps of the acoustic spectrum $E^-(k)$ and the optical spectrum $E^+(k)$ respectively.

By minimizing the free energy obtained from equations (6)–(8) at finite temperatures, we end up with self-consistent equations for the eight parameters of A, C, D, λ_A, λ_B, θ_A, θ_C and θ_D. Luckily, the three self-consistent equations about θ_A, θ_C and θ_D can be simplified by $\theta_A = \theta_C = \theta_D = 0$ or π, which means $\langle A_{2i,2i+1}\rangle$, $\langle C_{2i}\rangle$ and $\langle D_{2i+1}\rangle$ are real numbers. The simplified self-consistent equations are:

$$1 + 2S^A = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[\coth\left(\frac{E^-(k)}{2T}\right)\frac{\partial E^-(k)}{\partial\lambda_A} + \coth\left(\frac{E^+(k)}{2T}\right)\frac{\partial E^+(k)}{\partial\lambda_A}\right] \mathrm{d}k, \tag{10}$$

$$1 + 2S^B = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[\coth\left(\frac{E^-(k)}{2T}\right)\frac{\partial E^-(k)}{\partial\lambda_B} + \coth\left(\frac{E^+(k)}{2T}\right)\frac{\partial E^+(k)}{\partial\lambda_B}\right] \mathrm{d}k, \tag{11}$$

$$-8J_1A = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[\coth\left(\frac{E^-(k)}{2T}\right)\frac{\partial E^-(k)}{\partial A} + \coth\left(\frac{E^+(k)}{2T}\right)\frac{\partial E^+(k)}{\partial A}\right] \mathrm{d}k, \tag{12}$$

$$-2J_2C = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[\coth\left(\frac{E^-(k)}{2T}\right)\frac{\partial E^-(k)}{\partial C} + \coth\left(\frac{E^+(k)}{2T}\right)\frac{\partial E^+(k)}{\partial C}\right] \mathrm{d}k, \tag{13}$$

$$-2J_2D = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \left[\coth\left(\frac{E^-(k)}{2T}\right)\frac{\partial E^-(k)}{\partial D} + \coth\left(\frac{E^+(k)}{2T}\right)\frac{\partial E^+(k)}{\partial D}\right] \mathrm{d}k \tag{14}$$

where T is the reduced temperature (i.e. $k_B = 1$).

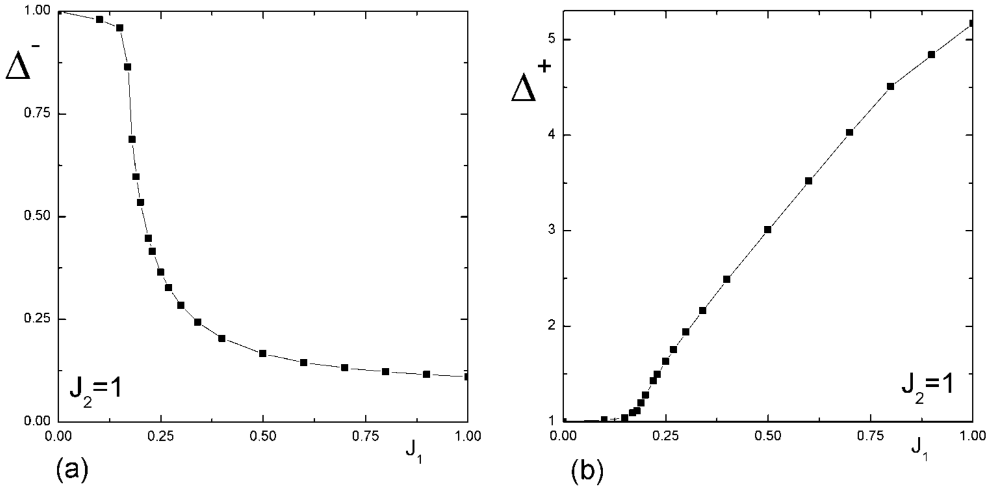


Figure 1. The energy gaps Δ^- and Δ^+ are plotted as functions of the interaction J_1 for $J_2 = 1$ in (a) and (b), respectively.

3. Numerical results and discussion

3.1. Properties at zero temperature

In order to investigate the effects of the interactions in the ladder system on the excitation spectrum, we start our discussion with two extreme cases.

First, we consider a simple limit case of $J_1/J_2 \sim 0$. Thus the spin ladder is decomposed into an array of two kinds of decoupled rungs, each rung representing a 'molecule' whose singlet ground state is separated from the triplet excited state by a large gap of the order of J_2:$\Delta^- = \Delta^+ = J_2$. It is shown that the low-lying excitation energy level is well represented by the SBMFT.

It is obvious that the two kinds of sublattice rung gaps between the ground state and the low-lying excitation state are the same at this limit of $J_1 = 0$, which means two gapful degenerate energy levels. When these sublattice rungs interact (i.e. let the coupling constant J_1 be finite), the original two degenerate energy levels will develop into two separate spectra: the acoustic spectrum $E^-(k)$ and the optical spectrum $E^+(k)$. Of course, the enhancement of the interaction between these sublattice rungs J_1 will lead to a decrease of the gap of the acoustic spectrum. This gap will disappear when J_1 increases to infinity, which is the other extreme case of a ferrimagnetic single chain which is discussed in the following. We calculated the two gaps within the regime of $0 < J_1 < 1$ at $J_2 = 1$, and plot them as a function of J_1 in figures 1(a) and (b). With increasing J_1, the acoustic gap decreases monotonically and the optical gap increases monotonically. When $J_1 = J_2 = 1$, we get $\Delta^- = 0.11$, which is smaller than the value obtained using the DMRG [8]: $\Delta^- = 0.33$.

Interestingly, one of the original degenerate energy levels goes down to develop the acoustic spectrum, and the other goes up to develop the optical spectrum when J_1 increases from 0 to 1 at $J_2 = 1$. At the same time, a finite bandgap appears between the acoustic spectrum and the optical spectrum. We choose some special case of J_1, and plot the two excitation spectra in figure 2. From this we know that the interactions of J_1 and J_2 in the system play an important role in determining the form of the excitation spectrum. For the acoustic spectrum, the width of the band increases monotonically with increase in J_1. But for

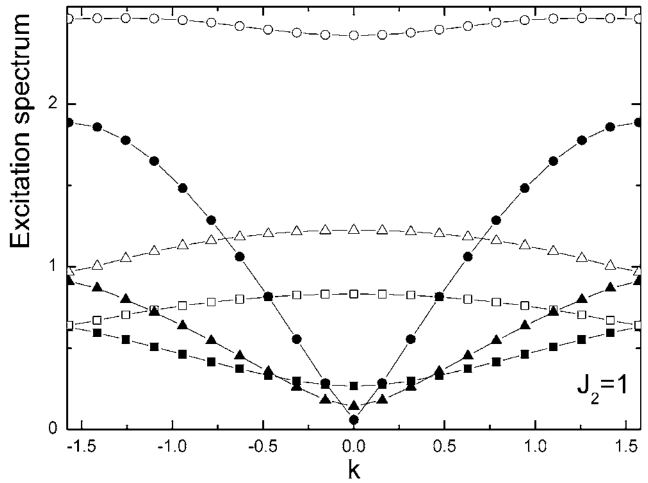


Figure 2. The excitation spectra of $E^-(k)$ and $E^+(k)$ are plotted as functions of k at $J_2 = 1$. The squares, triangles and circles correspond to the cases of $J_1 = 0.2$, 0.3 and 0.9 respectively. The full and open symbols denote the excitation spectra of $E^-(k)$ and $E^+(k)$ respectively.

the optical spectrum, the bandwidth is small and is hardly changed by the value of J_1, and its minimum position can be shifted by J_1. A bandgap appears between the two spectra at the verge of the Brillouin zone when the value of J_1/J_2 is smaller than 0.4. When $J_1/J_2 > 0.4$, the minimum of the optical spectrum shifts to the centre of the Brillouin zone.

Now we consider the other extreme case of a ferrimagnetic single chain at $J_2/J_1 = 0$, which has been proved to have a gapless acoustic spectrum and a gapful optical spectrum [9, 12]. When $J_2 = 0$, we can reduce our above formulae (7), (8) by $C = D = 0$ to the same form as that of [12, 13]. It should be taken into account that the Bogliubov quasi-particle n_k^- in equation (6) may undergo a condensation at absolute zero temperature because the excitation energy $E^-(k)$ has its minimal value $E^-(k = 0) = 0$ at $k = 0$. But no condensation happens for n_k^+ because of the finite optical gap. This has been discussed in detail in [12] and [13]. Of course, our results return to those.

When the two ferrimagnetic single chains interplay with an antiferromagnetic interaction, a gap opens up not only in the optical spectrum but also in the acoustic spectrum. We compute the two gaps Δ^- and Δ^+ in the regime of $0 < J_2 < 1$ for a fixed $J_1 = 1$, as shown in figures 3(a) and (b) respectively. We can see that both the acoustic gap and optical gap show a linear relation with the interaction between the two ferrimagnetic chains when the interaction J_2 is larger than 0.4, but a square relation in the weak interaction case. These conclusions are in good agreement with the results of the DMRG [8], which ensure us that the SMFT is effective in this mixed spin system.

The ratio of J_1 to J_2 also affects the excitation spectrum of the system at the same time. We choose three different cases of J_2, and plot the excitation spectrum in figure 4. At $J_2 = 0$, the system is reduced to the isolated ferrimagnetic chains, the gapless acoustic dispersion simply follows the k^2 dependence, which is denoted by square-type-line. When the two chains interact with a finite antiferromagnetic coupling interaction ($J_2 > 0$), the acoustic spectrum not only opens a gap but also displays a k linear relation near the centre of the first Brillouin zone. This is an important feature to enter into a gapful phase in many quantum spin systems, as pointed out by Fukui and Kawakami [7]. The two gapful acoustic spectra are marked by curves with full triangles and full circles for $J_2 = 0.6$ and 1 respectively. Apart from this,

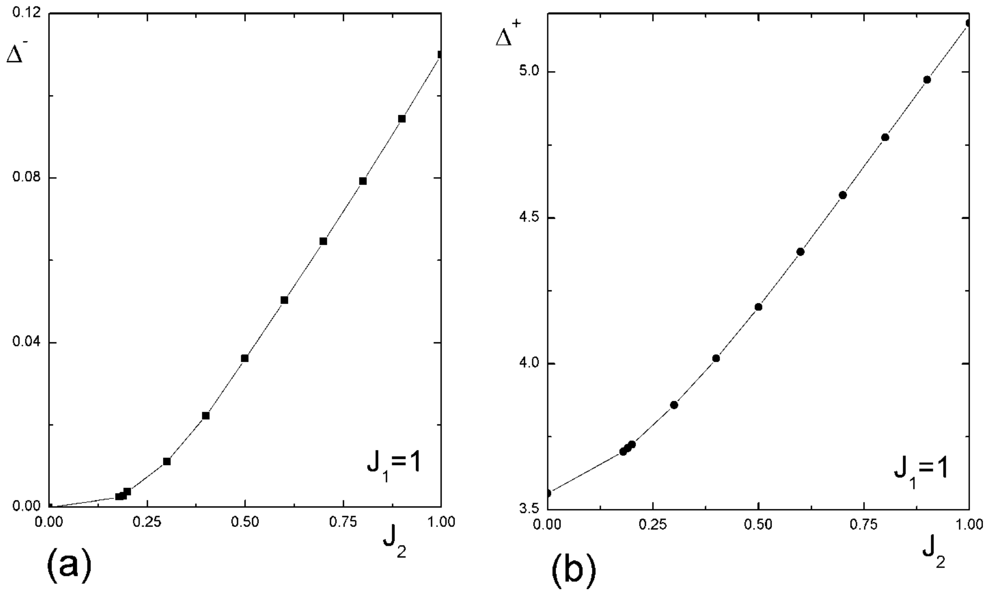


Figure 3. The energy gaps Δ^- and Δ^+ are plotted as functions of the interaction J_2 for $J_1 = 1$ in (a) and (b) respectively.

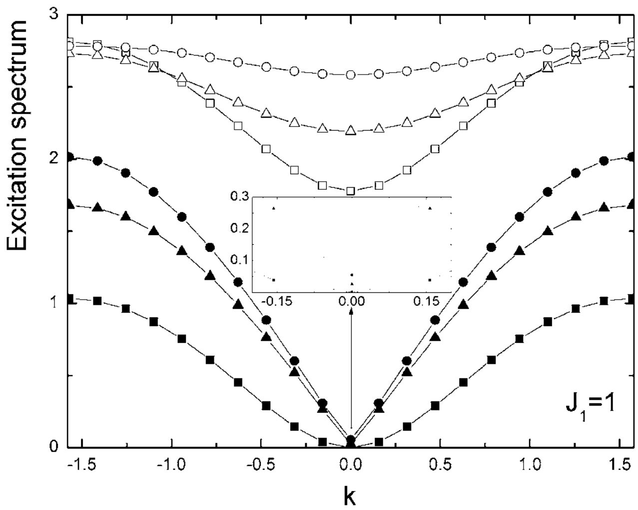


Figure 4. The excitation spectra of $E^-(k)$ and $E^+(k)$ are plotted as functions of k at $J_1 = 1$. The squares, triangles and circles correspond to the cases of $J_2 = 0$, 0.6 and 1. The full and open symbols denote the excitation spectra of $E^-(k)$ and $E^+(k)$ respectively.

the increase in the coupling constant J_2 enhances the bandwidth of the acoustic spectrum but reduces the bandwidth of the optical spectrum. This phenomenon is also observed in two-dimensional ferrimagnetic multichain systems [13], which is a sign that the one-dimensional two-leg ladder displays a crossover to a two-dimensional case.

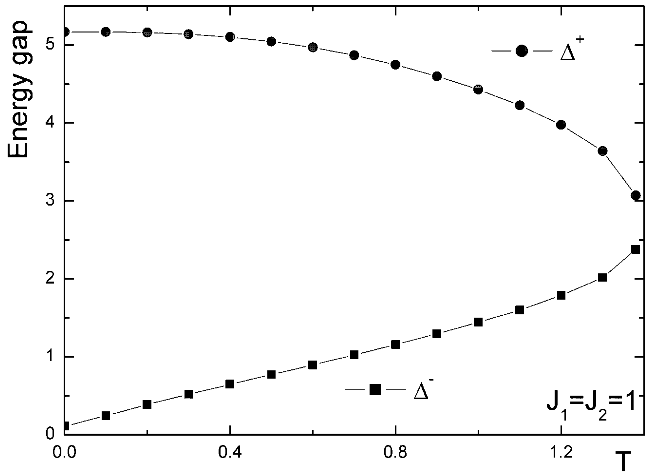


Figure 5. The energy gaps of Δ^- and Δ^+ are plotted as functions of the temperature for the special case of $J_1 = J_2 = 1$.

3.2. Properties at finite temperatures

Now we move to the thermodynamic properties at finite temperatures, and here we only consider the special case of $J_1 = J_2 = 1$. At low temperatures, the self-consistent equations (10)–(14) have meaningful solutions, but no solutions for high temperatures. This means that the SBMFT becomes invalid, as pointed out in [12, 13]. Thus we calculate the two energy gaps and the per site specific heat C_V at low temperatures.

We obtain the two gaps at low temperatures and show them as a function of temperature T in figure 5. As the temperature increases, the two gaps vary in different ways: the acoustic gap becomes bigger and the optical gap becomes smaller. This is a universal phenomenon in a mixed-spin system regardless of the dimension of the system. We have also observed it in both a one-dimensional mixed-spin system [12] and a two-dimensional mixed-spin system [13].

When the temperature is increasing, the bandwidths of the two excitation spectra both become smaller. The bosonic quasi-particle distribution confirmed the low-temperature trends to the classic double energy level distribution, which would make the high-temperature specific heat in a mixed-spin system turn on a Schottky-type peak as $C_V \propto (\frac{\Delta}{2k_B T} \sec h(\frac{\Delta}{2k_B T}))^2$ [14, 15], where Δ is the energy-level splitting. Although SBMFT predicts the trend to a double energy level structure at high temperatures, it misses the well-informed peak at mid temperatures, since it breaks down as we mentioned above.

At low temperatures, the per-site specific heat C_V versus temperature T is calculated by numerical differentiation of the internal energy with respect to T, and is shown in figure 6. Owing to the acoustic gap in the ground state, it shows an exponential relation with temperature, which is obviously different from the power law of $T^{1/2}$ in a ferrimagnetic single chain [12].

4. Conclusions

In summary, the low-lying excitation spectra in a system of quantum mixed-spin double ferrimagnetic chains are investigated in detail. We discuss the excitation spectra for various combinations of the two coupling parameters J_1 and J_2. In some special cases our results agree with earlier ones, which provide a good physical picture. For intermediate coupling strengths

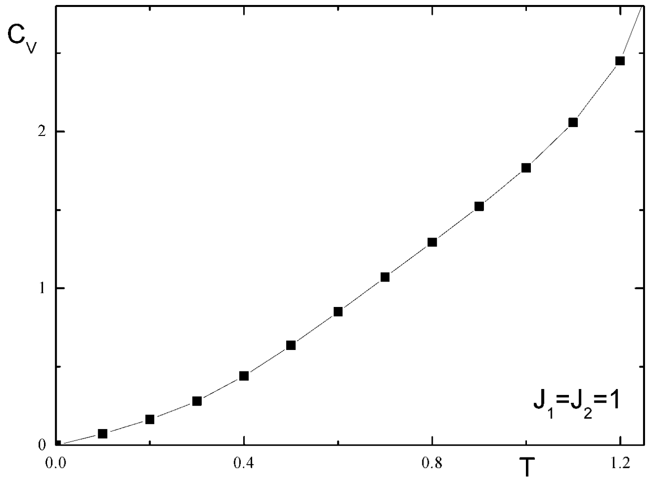


Figure 6. The per-site specific heat C_V plotted as a function of the temperature for the interaction couplings $J_1 = J_2 = 1$.

we also get reasonable results. We get a gap $\Delta^- = 0.11$, which is good enough to recall the generalization of Haldane's suggestions about antiferromagnetic coupling in ferrimagnetic systems, although it is smaller than the DMRG result of 0.33. As pointed out by Trumper *et al* [8], for a ferrimagnetic multichain any odd number of chains always has a ferrimagnetic ground state which is gapless but ordered and any even number of chains has a spin-gap behaviour analogous to the uniform spin-1/2 case. This paper gives a rigorous proof of this. The interaction between chains is mainly attributable to gap opening in the acoustic branch. The ground-state energy gap and the bandgap have some special effects on the magnetization curve at $T = 0$ K, to which we are giving further consideration.

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